Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MM04

Unit Mechanics 4

Friday 21 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

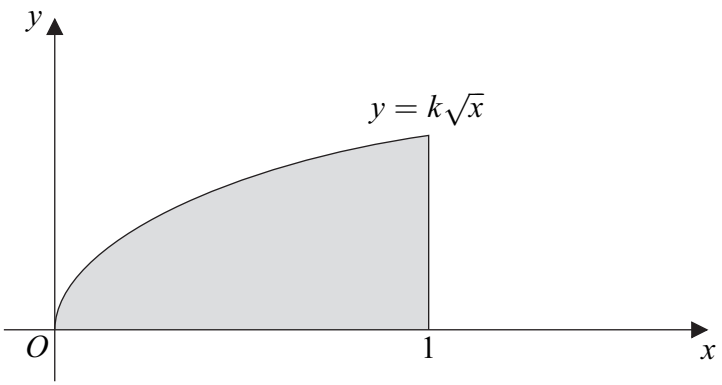
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A uniform lamina is bounded by the positive x-axis, the line $x = 1$ and the curve with equation $y = k\sqrt{x}$, where $k > 0$. The lamina is shown in the diagram.



The area of the lamina is A square units.

(a) Show that the x-coordinate of the centre of mass of the lamina is $\frac{2k}{5A}$. *(3 marks)*

(b) Find, in terms of k and A, the y-coordinate of the centre of mass of the lamina. *(3 marks)*

(c) Given that the centre of mass of the lamina lies on the line $y = x$, find the value of k. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 Three forces, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 5 \\ -4 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ a \end{bmatrix}$ newtons, act at the points with coordinates $(3, 4)$, $(-2, -3)$ and $(2, 0)$ metres respectively.

This system of three forces is equivalent to a force, $\mathbf{F}$, acting at the origin together with a couple of magnitude 24 Nm.

(a) Determine the two possible values for a. *(6 marks)*

(b) Write down the force, $\mathbf{F}$, corresponding to each value of a. *(2 marks)*

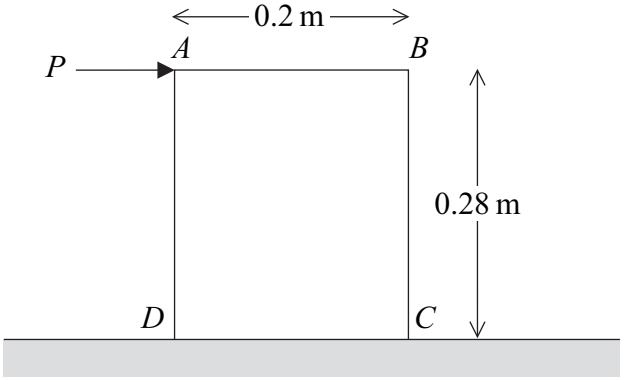
QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

3 A box of breakfast cereal, of total mass 0.4 kg, rests on a rough horizontal plane. It can be assumed that the centre of the mass of the box with its contents is on a vertical line through the centre of the box. The diagram shows a vertical cross section $ABCD$ through the centre of mass of the box and its contents, where $AB = 0.2\,\text{m}$ and $BC = 0.28\,\text{m}$. A horizontal force of magnitude P is applied at A, in the direction of AB.



The coefficient of friction between the box and the plane is μ. As P is gradually increased from zero, the box slides before it topples if, and only if, $\mu < k$. Find the value of k. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 3

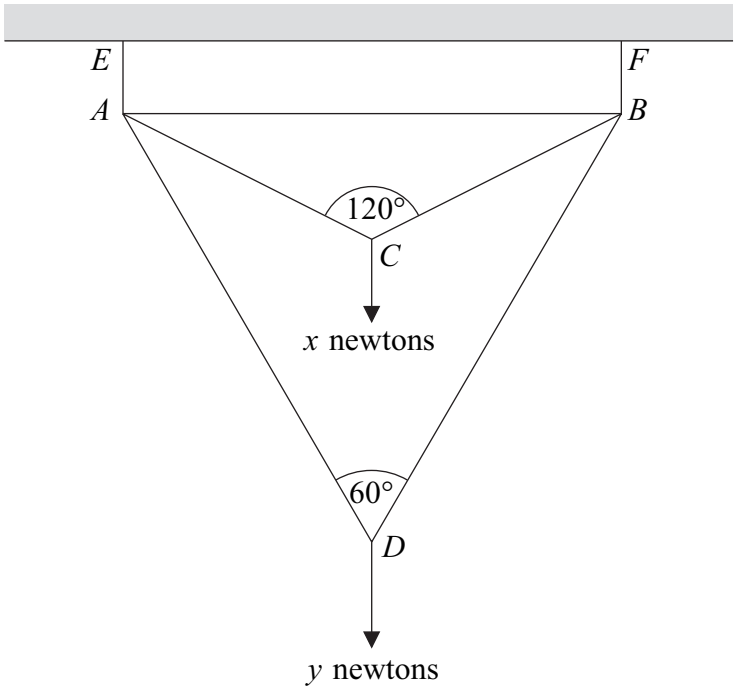
QUESTION PART REFERENCE

Answer space for question 3

4 A framework $ABCD$ consists of five light rods AB, AC, BC, AD and BD, which are smoothly jointed at A, B, C and D. The angle ACB is 120° and the angle ADB is 60°.

The framework is suspended from the fixed points E and F by strings EA and FB respectively. The rod AB is horizontal and D is vertically below C. The framework has a line of symmetry through C and D.

A force of x newtons is applied vertically downwards at C and a force of y newtons is applied vertically downwards at D. The system is in equilibrium.



The forces in the rods AC, AD, BC and BD are equal in magnitude.

(a) Show that $x : y = 1 : \sqrt{3}$. *(4 marks)*

(b) The tensions in the strings EA and FB are both 100 N.

(i) Find the magnitude of the force in rod AC. *(3 marks)*

(ii) Find the magnitude of the force in rod AB, stating whether the rod is in tension or compression. *(3 marks)*

QUESTION PART REFERENCE

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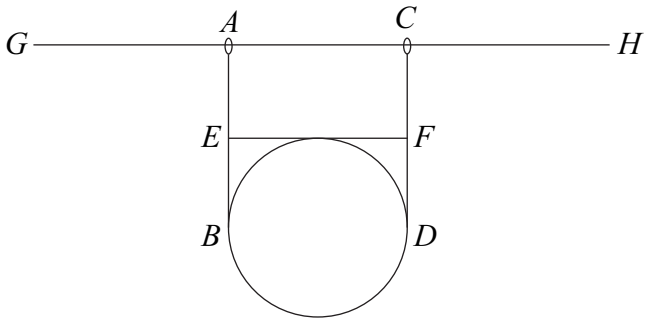
5 (a) A uniform circular disc has mass $20m$ and radius a.

(i) Prove, by integration, that the moment of inertia of the disc about an axis through its centre and perpendicular to the plane of the disc is $10ma^2$. *(5 marks)*

(ii) Hence determine the moment of inertia of the disc about a diameter, stating clearly any theorems that you use. *(3 marks)*

(b) A shop sign consists of a uniform circular disc of mass $20m$ and radius a, which is rigidly fixed to three rods AB, CD and EF. Each rod is of mass $2m$ and length $2a$. The rods are attached so that BD is a diameter of the disc; EF is a tangent to the disc, with A vertically above E and C vertically above F. The sign is suspended on a fixed horizontal pole GH by means of two small light rings which are attached to the sign at A and C, as shown in the diagram.



The sign can swing freely about the pole GH.

Find, in terms of m and a, the moment of inertia of the sign about GH. *(6 marks)*

QUESTION PART REFERENCE

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6 Four forces act at the points with coordinates as listed below.

Force		Point
$2\mathbf{i} + 3\mathbf{j}$	acts at	$A(1, 0, 0)$
$2\mathbf{i} - 3\mathbf{j}$	acts at	$B(1, 1, 0)$
$-\mathbf{j} + 2\mathbf{k}$	acts at	$C(0, 0, 1)$
$\mathbf{j} + 2\mathbf{k}$	acts at	$D(0, 1, 1)$

(a) Find the resultant of the four forces. *(1 mark)*

(b) Find the total moment of the four forces about O. *(5 marks)*

(c) An additional force **P** acts at a point on the y-axis so that the five forces are in equilibrium.

Determine the coordinates of the point on the y-axis at which **P** acts. *(6 marks)*

QUESTION PART REFERENCE

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7 A system consists of a uniform rod AB, of mass $3m$ and length $8a$, and a particle of mass m attached to the end B. The system can rotate freely about a fixed horizontal axis perpendicular to the rod AB. The axis passes through the point C, the mid-point of AB. The centre of mass of the system is G.

(a) Show that $CG = a$. *(2 marks)*

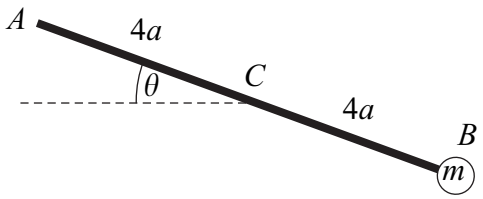
(b) Show that the moment of inertia of the system about the axis through C is $32ma^2$. *(3 marks)*

(c) The system is held at rest in a horizontal position, as shown in **Figure 1**.

Figure 1

The system is released from rest and begins to rotate. At time t, the system has turned through an angle θ, as shown in **Figure 2**.

Figure 2



(i) Show that the angular speed of the system at time t is

$$\sqrt{\frac{g\sin\theta}{4a}}$$

(4 marks)

(ii) Find an expression, in terms of a, g and θ, for the angular acceleration of the system. *(2 marks)*

(iii) The component of the reaction force perpendicular to the rod at C is Y.

The component of the reaction force parallel to the rod at C is X.

Find X and Y in terms of m, g and θ. *(6 marks)*

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